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Assignment: Calculus I Quiz 1

Chapter 1, Section 1.1, Question 11b

Find the natural domain of the function algebraically, and confirm that your result is consistent with the graph produced by your graphing utility.

$$y = \sqrt{x^2 - 64}$$

- The natural domain of the function is $(-\infty, -8] \cap [8, \infty)$.
- The natural domain of the function is $[8, \infty)$.
- The natural domain of the function is $(-\infty, -8] \cup [8, \infty)$.
- The natural domain of the function is $(-\infty, -8]$.
- The natural domain of the function is $(-\infty, -64] \cup [64, \infty)$.

Answer: **C. The natural domain of the function is (-∞, -8] ∪ [8, ∞).**

Solution: Think about the function and the definition of a natural domain. Many times, logic can be used to determine limits and domains.

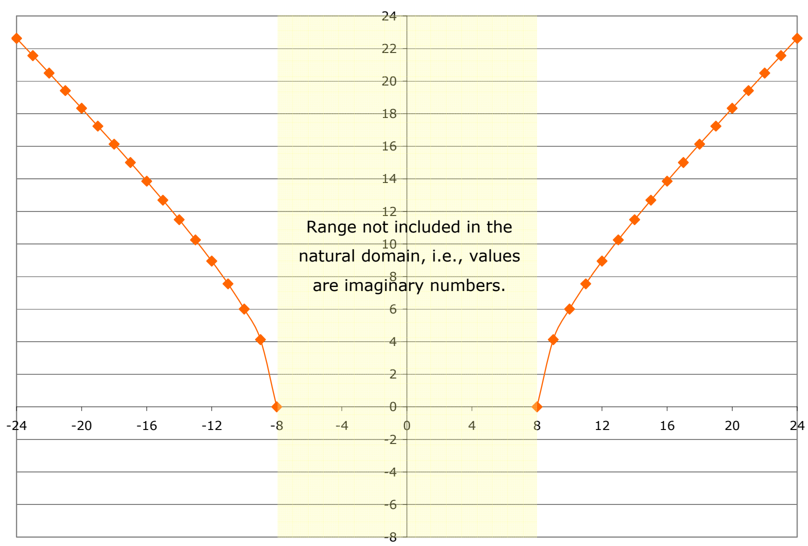
If a function is defined by a formula where there is no domain explicitly stated, it is understood that the domain consists of all real numbers that where the formula makes sense, and the function has a real value. This is called the natural domain of the function.

Negative values under a radical sign give imaginary numbers. Because we are looking for the natural domain of the function, the expression under the radical sign can not be negative, i.e,

$$x^2 - 64 \geq 0$$

Solving the inequality gives

$$x^2 - 64 \geq 0$$
$$x^2 \geq 64$$
$$(x - 8)(x + 8) \geq 64$$
$$x \geq 8 \text{ and } x \leq -8$$



The inequality has two solutions because squaring a negative number gives a positive number. Therefore, the set of numbers that give a positive value under the radical sign in this function above is -∞ to -8 and 8 to ∞.

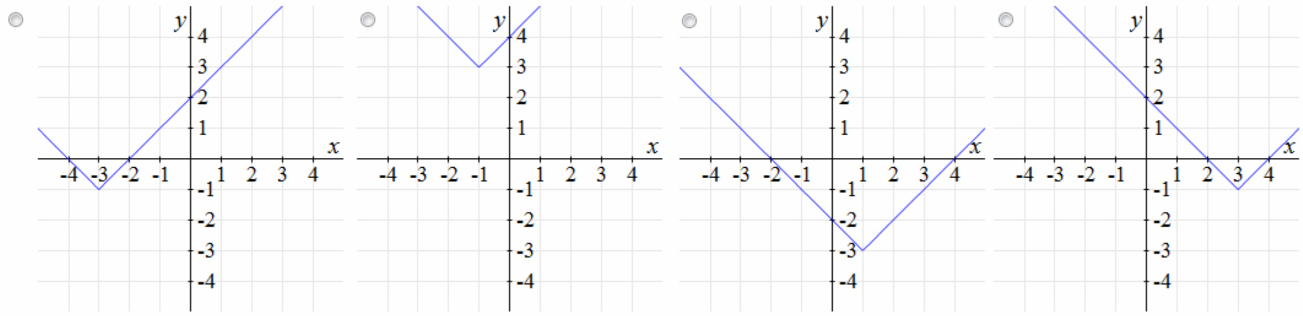
The next step is to understand the symbols used to express the domain set. *Brackets* [] indicate closed intervals, meaning the numbers are included in the set. *Parenthesis* () indicate open intervals, meaning the numbers are not included in the set. The union (open up) ∪ indicates a *union* of two disjointed/unconnected sets (which do not have numbers in common) and results in a single set containing all elements from both disjointed/unconnected sets. The union (open down) ∩ indicates an *intersection* of two sets where the final set contains only numbers in common from both sets.

Therefore, the answer is (-∞, -8] ∪ [8, ∞) because both 8 and -8 are give real values for the function and are included in the set and infinity (both negative and positive) are open ended values.

To confirm the answer, graph the function.

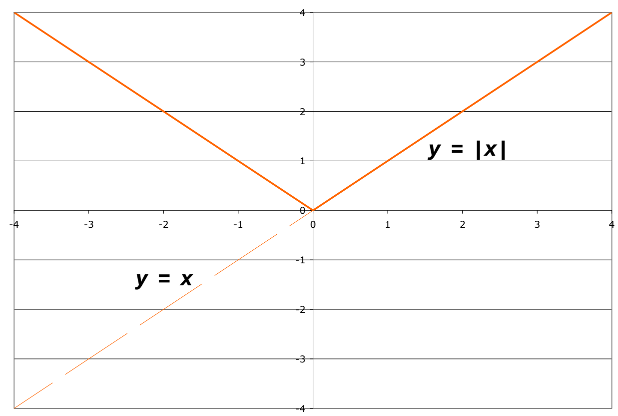
Chapter 1, Section 1.3, Question 19

Sketch the graph of the equation $y = |x + 3| - 1$ by translating, reflecting, compressing, and stretching the graph of $y = |x|$ appropriately, and then use a graphing utility to confirm that your sketch is correct.



Answer: **A.**

Solution: We will solve this in a stepwise fashion and confirm with graphing.



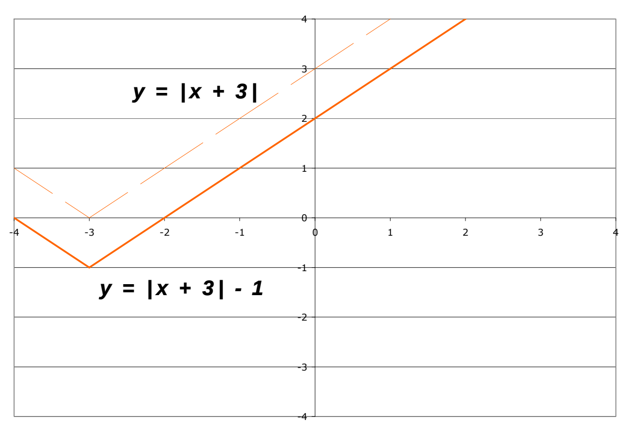
The first step is to start with the expression y = |x| which is simply a modified line y = x with no negative numbers.

First, sketch the graph y = x, which is a basic line. Modify the y = x line by eliminating the negative numbers. This means that y ≥ 0, i.e., no negative values for y exist in the expression y = |x|.

The resulting graph looks like the letter "V" with the point on zero.

The second step is to modify the y = |x| by adding + 3 *inside* the absolute value sign.

It is very important to note that the addition of 3 occurs inside the absolute value sign. The easiest way to evaluate the expression y = |x + 3| is by separating the positive and negative ranges. We know that at that point, the slope of the line changes. The point where y = 0 is x = -3. Therefore, the point of the graph shifts 3 units to the left. Sketch the graph by shifting the line.



The third step is to modify the expression y = |x + 3| by subtracting 1.

The subtraction of 1 occurs outside of the absolute value sign, so it singles only a downward shift of the entire line 1 unit.

Confirm the answer by graphing the expression.

Chapter 1, Section 1.6, Question 29

Solve for x without using a calculating utility. Use the natural logarithm anywhere that logarithms are needed.

$$4e^{6x} = 10 \,.$$

Carry out all calculations exactly and round to 2 decimal places the final answer only.

$x =$ ______ .

the tolerance is +/-2%

Answer: **0.15**

Solution: Solve the equation remembering the following relationships:

$$y = b^x \text{ is equivalent to } log_b(y) = x$$

$$log_e(y) \text{ is equivalent to } ln(y)$$

The solution is

$$\begin{aligned} 4e^{6x} &= 10 \\ e^{6x} &= \frac{10}{4} \\ e^{6x} &= \frac{10}{4} \\ 6x &= log_e\left(\frac{10}{4}\right) \\ 6x &= ln\left(\frac{10}{4}\right) \end{aligned}$$

$$x = \frac{ln\left(\frac{10}{4}\right)}{6} = 0.1527 \approx 0.15$$

Chapter 1, Section 1.6, Question 43b

Find the equation of an exponential function that passes through the point $(16, 2)$.

- $y = 4^x$
- $y = \left(\sqrt{2}\right)^x$
- $y = \left(\frac{1}{4}\right)^x$
- $y = \left(\sqrt[16]{2}\right)^x$
- $y = \left(\frac{1}{\sqrt[16]{2}}\right)^x$

Answer: **D.** $y = \left(\sqrt[16]{2}\right)^x$

Solution: Using logic, we can eliminate choice A because we can not raise a positive integer by another positive integer and get a smaller number. Choice B is similar to Choice A; even though $\sqrt{2} < 5$, $x \gg y$. Choices C and E seem unlikely to result in an integer, i.e., y = 2. We guess that the answer is choice D, but we will work out each expression to confirm the answer.

<u>Choice A:</u> $y = 4^x$ where (16,2) or x = 16 and y = 2

$$y = 4x \quad \Rightarrow \quad (2) = 4^{(16)} \quad \Rightarrow \quad 2 \neq 4.29 \times 10^9$$

<u>Choice B:</u> $y = \left(\sqrt{2}\right)^x$ where (16,2) or x = 16 and y = 2

$$y = \left(\sqrt{2}\right)^x \quad \Rightarrow \quad (2) = \left(\sqrt{2}\right)^{(16)} \quad \Rightarrow \quad 2 = 2^{\frac{16}{2}} \quad \Rightarrow \quad 2 \neq 2^8$$

<u>Choice C:</u> $y = \left(\frac{1}{4}\right)^x$ where (16,2) or x = 16 and y = 2

$$y = \left(\frac{1}{4}\right)^x \quad \Rightarrow \quad (2) = \left(\frac{1}{4}\right)^{(16)} \quad \Rightarrow \quad 2 \neq 2.33 \times 10^{-10}$$

<u>Choice D:</u> $y = \left(\sqrt[16]{2}\right)^x$ where (16,2) or x = 16 and y = 2

$$y = \left(\sqrt[16]{2}\right)^x \quad \Rightarrow \quad (2) = \left(\sqrt[16]{2}\right)^{(16)} \quad \Rightarrow \quad 2 = 2^{\frac{16}{16}} \quad \Rightarrow \quad 2 = 2$$

<u>Choice E:</u> $y = \left(\frac{1}{\sqrt[16]{2}}\right)^x$ where (16,2) or x = 16 and y = 2

$$y = \left(\frac{1}{\sqrt[16]{2}}\right)^x \Rightarrow (2) = \left(\frac{1}{\sqrt[16]{2}}\right)^{(16)} \Rightarrow 2 = \left(\frac{1}{2^{\frac{1}{16}}}\right)^{(16)} \Rightarrow 2 = \left(2^{-\frac{1}{16}}\right)^{(16)} \Rightarrow 2 = 2^{-\frac{16}{16}} \Rightarrow 2 = 2^{-1} \Rightarrow$$

$$2 \neq 0.5$$

Chapter 2, Section 2.1, Question 13a

Make a guess at the limit (if it exists) by evaluating the function at the specified *x*-values.

$$\lim_{x \to 2} \frac{x-2}{x^3-8}; \quad x = 3, 2.5, 2.1, 2.01, 2.001, 1, 1.5, 1.9, 1.99, 1.9990$$

- the limit doesn't exist
- 0
- 1
- $\frac{1}{8}$
- $\frac{1}{12}$

Answer: **E.** $\mathbf{\frac{1}{12}}$

Solution: Upon inspection, we know this expression is not valid for $x = 2$ because the denominator would be 0. As a guess, we will assume that those values of *y* when *x* approaches 2 would be the limit. To estimate, we will assume that an *x* close to 2, but not equal to 2 would yield the following

numerator: $x - 2$ where x is close to 2, but not equal to 2, i.e., $x \approx 2$
$(\sim 2) - 2 =$ a number that is a fraction/decimal

If we estimate an x = 1.9, the numerator would be 0.1 or the fraction, $-\frac{1}{10}$.

denominator: $x^3 - 8$ where x is close to 2, but not equal to 2, i.e., $x \approx 2$
$(\sim 2)^3 - 8 =$ a number that is a fraction/decimal

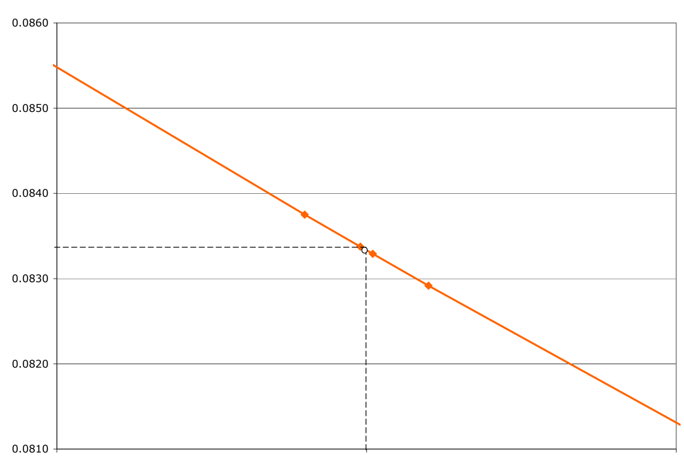
If we estimate an x = 1.9, the denominator would be $\left(\frac{19}{10}\right)^3 - \left(\frac{8}{1}\right) \Rightarrow \frac{\sim 6800}{1000} - \frac{8000}{1000} \Rightarrow -\frac{1200}{1000}$

Putting the numerator and denominator together yields

$$\frac{-\frac{1}{10}}{-\frac{1200}{1000}} \Rightarrow \frac{-1 \times 1000}{-10 \times 1200} \Rightarrow \frac{-1000}{-12000} \Rightarrow \frac{1}{12}$$

We guess a limit of $\frac{1}{12}$ and then evaluate the functions at the specified *x*-values to confirm the answer.

x	*y*
3	0.0526
2.5	0.0656
2.1	0.0793
2.01	0.0829
2.001	0.0833
1.9990	0.0834
1.99	0.0838
1.9	0.0876
1.5	0.1081
1	0.1429



Chapter 2, Section 2.2, Question 3

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Find the limit.

$$\lim_{x \to 8} x(x-1)(x+1)$$

Answer: **504**

Solution: Make a guess and then evaluate *x*-values near 8.

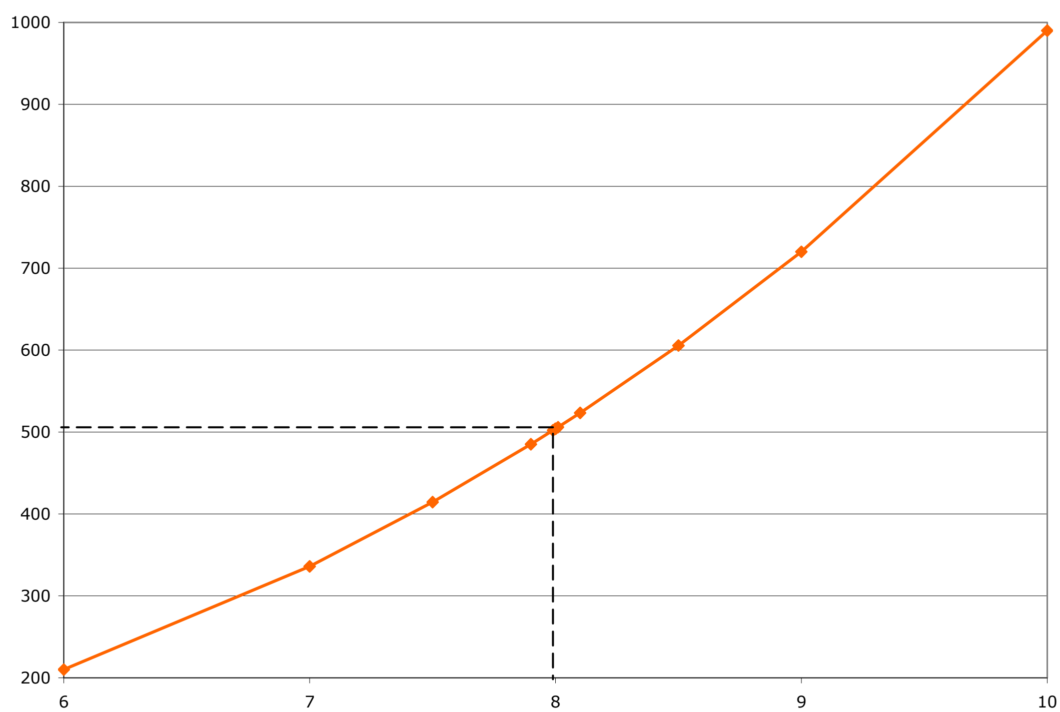
The function f(x) = $x(x - 1)(x + 1)$, solve for $x = 8$.

$$f(8) = 8(8 - 1)(8 + 1) = 8(7)(9) = 504$$

Next, solve for *x*-values near 8, approaching from the positive and negative sides.

x	*y*
6	210
7	336
7.5	414.38
7.9	485.14
7.99	502.09
7.999	503.81
8.001	504.19
8.01	505.91
8.1	523.34
8.5	605.63
9	720
10	990

Remember, a limit does not necessarily mean the function is not valid at that point.



Chapter 2, Section 2.2, Question 19

Find the limit

$$\lim_{x \to 2^-} \frac{x}{x^2 - 4}$$

- $\frac{1}{2}$
- $+\infty$
- 2
- $-\infty$
- $-\frac{1}{2}$

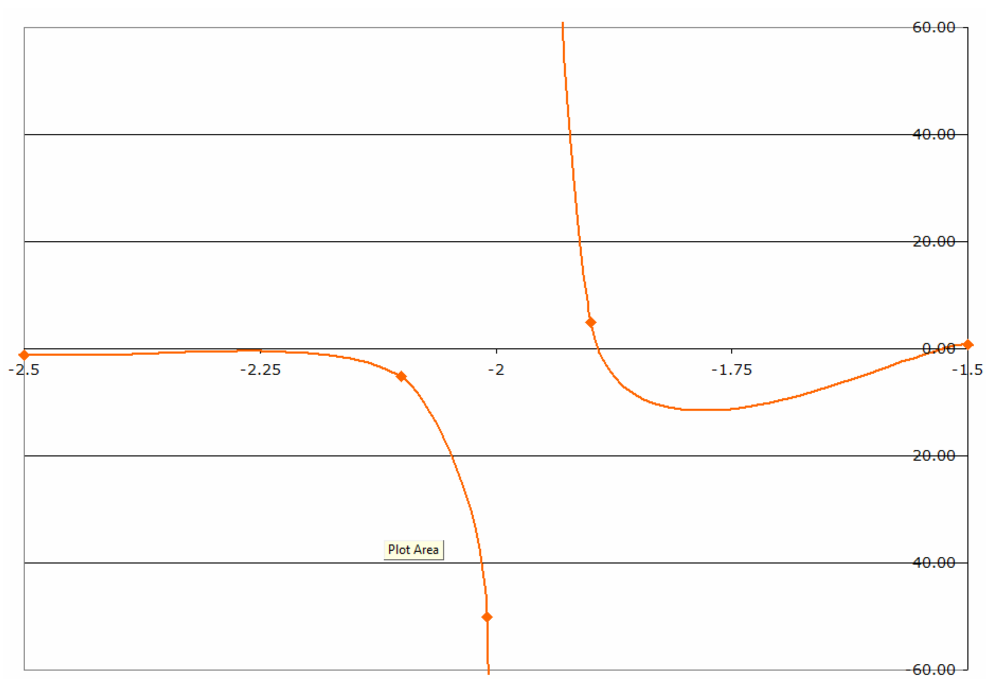
Answer: **D. -∞**

Solution: Make a guess and then evaluate *x*-values near 2, approaching from the negative side.

The function $f(x) = \frac{x}{x^2 - 4}$ is a parabolic function, so we will guess that the limit will be infinity.

Next, solve for *x*-values near 2, approaching from the negative side.

x	***y***
-4	-0.33
-3	-0.6
-2.5	-1.11
-2.1	-5.12
-2.01	-50.12
-2.001	-500.12
-1.9	4.87
-1.5	0.86
-1	0.33
0	0



Chapter 2, Section 2.2, Question 23

Find the limit.

$$\lim_{y \to 4} \frac{y+4}{y^2-16}$$

- $+\infty$
- doesn't exist
- $-\infty$
- 1
- 4

Answer: **B. doesn't exist**

Solution: Normally, to solve a limit, we would make a guess and then evaluate *y*-values near 4, approaching from the positive and negative sides. In addition, we know that at $y = \pm 4$, this function is not valid. However, it is important to remember the limit theorems that may be applicable. It does not make sense to look for a limit if one does not exist.

One theorem of limits states the following relationship

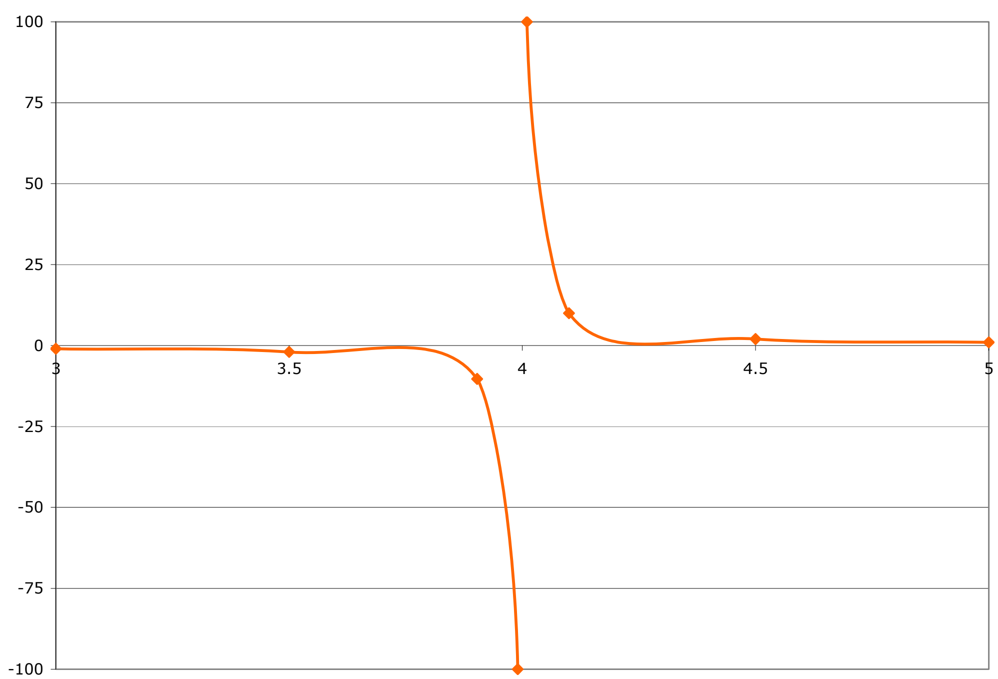
$$\text{where } \lim_{x \to c} f(x) = L \text{ and } \lim_{x \to c} g(x) = M,\ \lim_{x \to c} \frac{f(x)}{g(x)} = \begin{cases} \frac{L}{M}, & \text{if } M \neq 0 \\ \text{does not exist, if } M = 0 \text{ and } L \neq 0 \end{cases}$$

In our function,

$$f(x) = y + 4 \text{ and } \lim_{x \to c} f(x) = L = (4) + 4 = 8$$

$$g(x) = y^2 - 16 \text{ and } \lim_{x \to c} g(x) = M = (4)^2 - 16 = 0$$

The second condition of the theorem where $M = 0$ and $L \neq 0$ exists. Therefore, the limit does not exist.



Chapter 2, Section 2.2, Question 23

Find the limit.

$$\lim_{y \to 8} \frac{y+8}{y^2-64}$$

- doesn't exist
- 8
- $-\infty$
- $+\infty$
- 1

Answer: **A. doesn't exist**

Solution: Normally, to solve a limit, we would make a guess and then evaluate *y*-values near 8, approaching from the positive and negative sides. In addition, we know that at $y = \pm 8$, this function is not valid. However, it is important to remember the limit theorems that may be applicable. It does not make sense to look for a limit if one does not exist.

One theorem of limits states the following relationship

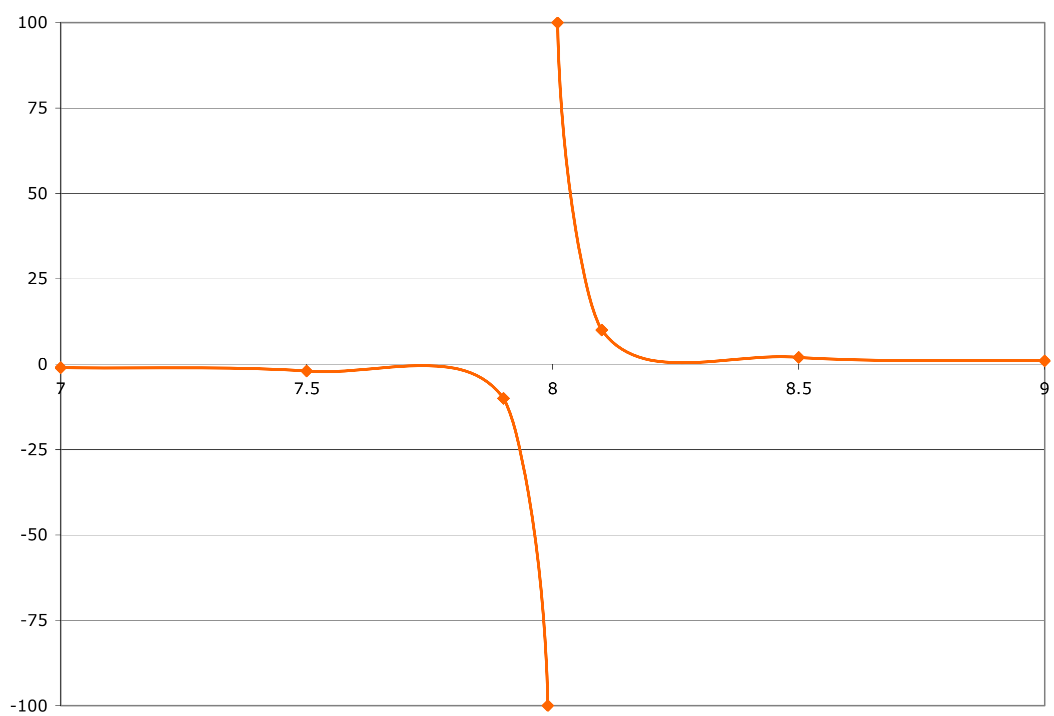
$$\text{where } \lim_{x \to c} f(x) = L \text{ and } \lim_{x \to c} g(x) = M, \ \lim_{x \to c} \frac{f(x)}{g(x)} = \begin{cases} \frac{L}{M}, & \text{if M} \neq 0 \\ \text{does not exist}, & \text{if M} = 0 \text{ and L} \neq 0 \end{cases}$$

In our function,

$$f(x) = y + 8 \text{ and } \lim_{x \to c} f(x) = L = (8) + 8 = 16$$

$$g(x) = y^2 - 64 \text{ and } \lim_{x \to c} g(x) = M = (8)^2 - 64 = 0$$

The second condition of the theorem where $M = 0$ and $L \neq 0$ exists. Therefore, the limit does not exist.



Chapter 2, Section 2.2, Question 37

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First rationalize the numerator, then find the limit.

$$\lim_{x \to 0} \frac{\sqrt{x+49}-7}{x}$$

Write answer in the form of simple fraction, if it is necessary.

Answer: $\mathbf{\frac{1}{14}}$

Solution: First, rationalize the numerator

$$\frac{\sqrt{x+49}-7}{x} \bullet \frac{\sqrt{x+49}+7}{\sqrt{x+49}+7}$$
$$\frac{x+49-49}{x\left(\sqrt{x+49}+7\right)}$$
$$\frac{x}{x\left(\sqrt{x+49}+7\right)}$$

Therefore, the expression can be simplified to

$$\frac{1}{\sqrt{x+49}+7}$$

For $x = 0$

$$\frac{1}{\sqrt{0+49}+7} = \frac{1}{7+7} = \frac{1}{14}$$